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THIN VISCOUS ELLIPTICAL ACCRETION DISCS WITH ORBITS SHARING A COMMON LONGITUDE OF PERIASTRON. DYNAMICAL EQUATION FOR INTEGER VALUES OF THE POWERS IN THE VISCOSITY LAW

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Abstract

We consider a model of thin stationary viscous accretion disc around a stellar mass compact object developed by Lyubarskij et al. [3]. The orbits of the gaseous particles are ellipses which eccentricities may vary from inner to the outer parts of the disc and which apse lines are in line with each other. The accepted viscosity coefficient η *obeys the relation* $\eta = \beta \Sigma^n$ *with* Σ *- surface density of the accretion disc,* β *and n - constants. Our considerations are dealing with the cases when the exponent n takes integer values, namely* $n = -1, 0, 1, 2$ *and 3, which lie in the physically suitable range. We derive in an explicit form the auxiliary functions introduced by Lyubarskij et al. For two values of* $n = -1$ *and* $n = +2$ *we also write the explicit form of the dynamical equation governing the radial structure of the disc. For the other cases we limit us with graphical representations of the ratios of the coefficients of this equation.*

Keywords: *accretion discs.*

1. Introduction

There are both observational and theoretical grounds to believe that the circular orbits of fluid particles in accretion discs are not the only possible cases which may be considered in treatment of accretion phenomena. The most widespread applications of using eccentric orbits for description and explanation of the observed astrophysical events are the superhumps in the light-curves of dwarf-nova cataclysmic variables like SU UMa stars. This type of binary stars consists of a white dwarf, with a gaseous accretion disc around it, and a main sequence star which supplies matter to the disc through the inner Lagrangian point L_1. The internal instability of the disc, caused by viscosity stresses, as well as the tidal influence of the companion star, are the reasons determining the elongated (elliptical) shape of the disc. Its dimensions also vary during different stages of the outburst events depending on the total accumulated mass and the thermal conditions. The mass transfer stream from the companion star strikes the outer parts of the accretion disc at the so-called "hot-spot" region. But nevertheless, it is not expected the dynamics of the accretion disc to be significantly affected by that perturbation. For example, time-resolved spectroscopy is applied to study the nova-like variable UU Aqu. Using eclipse mapping techniques, spatially resolved spectra of its accretion disc as a function of the distance from the disc centre were obtained. Consideration of the data suggests that the asymmetric structure in the outer disc (previously identified as a bright spot) may be considered as a signature of an elliptical disc, similar to those in SU UMa stars during superoutbursts [1]. However, it is worth noting that this interpretation is not the only possible one. The non-axisymmetric features observed in the discs of dwarf-novae during the outburst events are often considered to be spiral shocks, but this explanation strikes with some problems: the natural site of the wave excitation lies outside the Roche lobe, the accretion disc must be "hot", the treatments of wave propagation does not take into account the vertical disc structure [2]. Consequently, the elliptical shape of the discs in these cases remains a plausible explanation of the observed features of dwarf-nova outbursts.

During the recent years, increasing interest has been devoted to the problem of formation of planetary systems around solar-like young stellar systems. Here, the accretion discs, from which the planets generate, may consist not only of gaseous component, but be predominantly composed of solid dust particles and rocks, and have a complex radial structure. An accreting protoplanet that is embedded into the disc may clear an annulus about its orbital path. Numerous observational efforts have lead to the discovery of many extrasolar planet systems (the number of planets approaches one hundred at present time) and, in the majority of cases, the eccentricities of the planet orbits were evaluated with sufficient accuracy. These estimates definitely lead to the conclusion that, as a rule, the extrasolar planets have orbits with considerable eccentricities - evidence that the progenitor accretion discs were also with elliptical shape.

The large variety of possibilities for the parameters of the systems "accretion disc + binary star" suggests, in turn, a large number of theoretical models for these astrophysical systems. It is not always possible to solve analytically problems arising in this way and numerical approaches are needed to find the solutions of the equations describing the discs dynamics. In this paper, we focus on a model of elliptical accretion disc developed by Lyubarskij et al. [3]. Our aim is to obtain in explicit form the dynamical equation describing the properties of the accretion disc around a stellar mass compact object for some particular values of the viscosity law parameters. An important specific feature of this model is that the apse lines of all particle orbits are in line with each other. This condition, imposed "by hand", may be removed, as it has been done in more recent studies of fluid dynamics of eccentric discs by Ogilvie, by using complex values of disc eccentricity [4]. But this complication makes it much more difficult to find an analytical solution to the dynamical equation of the disc. Our main purpose is to use an analytical approach to the considered problem. We restrict ourselves to the more simple task based on the model of Lyubarskij et al. [3], although the accuracy of this description (in opposite to the model of Ogilvie [4]) is not enough suitable to make precise tests of the theory by observations. Nevertheless, we hope that the fully analytical treatment of the accretion flows in such simplified cases may be useful in the attempts to solve analytically (or to determine the limits of the analytical approach) the more complicated and realistic models of accretion discs, which are appropriate for evaluating the model parameters from direct comparison with observations.

2. Accretion Disc Model

In what follows, we shall use the notations and approach according to the paper of Lyubarskij et al. [3]. The eccentric disc model, considered in this paper, includes also the non-stationary regime, but we shall limit ourselves only to the stationary picture. The theory represents, to some extent, a generalization of the standard thin α-disc theory to the case of elliptical streamlines of gaseous particles. The accepted viscosity law describes a proportionality between the viscosity coefficient η and the n-th power of disc surface density Σ: $\eta = \beta\Sigma^n$, where β and n are constants. Our intention is to write explicitly and to investigate the possibility for exact analytical solution of the dynamical equation, governing the radial structure of the accretion disc, for *integer* values of the power n, namely for $n = -1, 0, 1, 2$, and 3. These selections are of astrophysical interest and the implications for non-integer values of n may possibly be obtained through an interpolation between the data for these integer numbers. In the considered model, the eccentricity e of the particle orbits may vary under the transition from the inner to the outer parts of the disc. For every elliptical orbit, the dependence of its eccentricity e on the focal parameter p ($p = b^2/a$; a and b are the major and the minor semiaxes), giving the "size" of the ellipse, is determined by the following dynamical equation [3]:

$$(1) \quad [\, Y\,(\partial Z/\partial \dot{e}) - Z\,(\partial Y/\partial \dot{e})\,]\,\ddot{e} + [\, Y\,(\partial Z/\partial e) - Z\,(\partial Y/\partial e) - Y^2 e\,]\,\dot{e} + Y\,[\,(3/2)W - Z - (1/2)(1 - e^2)\,Y\,] = 0 \,.$$

This is a second order ordinary differential equation, where the dot ($\cdot$) denotes differentiation with respect to the variable $u \equiv \ln p$ and it is taken into account that $e = e\,(p, n)$. The analytical expressions for auxiliary functions Y, Z and W (averaged over the azimuthal angle φ) and the integrals I_{0-}, I_{0+} and I_k ($k = 0, 1,\ldots,4$) are given in a previous paper [5], devoted to the investigation of equation (1). All these quantities are functions of e, $\dot{e} \equiv \partial e/\partial u$ and n. In the present study, we have computed in explicit form the integrals I_{0-}, I_{0+} and I_k ($k = 0, 1,\ldots,4$), and correspondingly Y, Z and W for the above mentioned integer values of the exponent n. This is done by the use of some already tabulated integrals ([6], formulae 858.525 and 858.535) and consequential application of the derived results for the next steps of the evaluations. We remind here that, according to Lyubarskij et al. [3], the negative values of the eccentricity e simply imply that the periastron of the ellipse lies on the negative part of the abscissa axis as opposite to the case of positive values of e, when its abscissa is positive. We stress again that the considered model of particle orbits includes only apse lines in line with each other, i.e., all ordinates of the periastron points are equal to zero. We obtain the following results:

<u>Case $n = -1$</u>

(2a) $I_0(e, \dot{e}, n = -1) = \pi\,(1 - e^2)^{-5/2}\,(2 + e^2)\,,$

(2b) $I_1(e, \dot{e}, n = -1) = -3\pi\, e\,(1 - e^2)^{-5/2}\,,$

(2c) $I_2(e, \dot{e}, n = -1) = \pi\,(1 - e^2)^{-5/2}\,(1 + 2e^2)\,,$

(2d) $I_3(e, \dot{e}, n = -1) = \pi\,(1 - e^2)^{-5/2}\, e^{-3}\,[-2 + 5e^2 - 6e^4 + 2(1 - e^2)^{5/2}\,]\,,$

(2e) $I_4(e, \dot{e}, n = -1) = 3\pi\,(1 - e^2)^{-5/2}\, e^{-4}\,[\,2 - 5e^2 + 4e^4 - 2(1 - e^2)^{5/2}\,]\,,$

(2g) $I_{0+}(e, \dot{e}, n = -1) = \pi\,(1 - e^2)^{-5/2}\,[1 - (e - \dot{e})^2\,]^{-1/2}\,\dot{e}^{-3}\,\{-2e\,(1 - e^2)\,\dot{e}\,(e - \dot{e})$? $[1 - (e - \dot{e})^2\,]^{1/2} + 2e\,(1 - e^2)^2\,(e - \dot{e})^2\,[1 - (e - \dot{e})^2\,]^{1/2} - 2(1 - e^2)^{5/2}\,(e - \dot{e})^3 + e\,(2 + e^2)\,\dot{e}^2\,[1 - (e - \dot{e})^2\,]^{1/2}\,\}\,,$

(2h) $I_{0-}(e, \dot{e}, n = -1) = \pi\,(1 - e^2)^{-7/2}\,(2 + 3e^2)\,.$

Case $n = 0$

(3a) $$I_0(e, \dot{e}, n=0) = 2\pi (1-e^2)^{-3/2} [1-(e-\dot{e})^2]^{-1/2} \dot{e}^{-2} \{ e\dot{e} [1-(e-\dot{e})^2]^{1/2} - e(e-\dot{e}) \; ? \; (1-e^2)[1-(e-\dot{e})^2]^{1/2} + (e-\dot{e})^2 (1-e^2)^{3/2} \},$$

(3b) $$I_1(e, \dot{e}, n=0) = 2\pi (1-e^2)^{-3/2} [1-(e-\dot{e})^2]^{-1/2} \dot{e}^{-2} \{ (e-\dot{e}-e^3)[1-(e-\dot{e})^2]^{1/2} - (e-\dot{e})(1-e^2)^{3/2} \},$$

(3c) $$I_2(e, \dot{e}, n=0) = 2\pi (1-e^2)^{-3/2} [1-(e-\dot{e})^2]^{-1/2} \dot{e}^{-2} \{ (-1+e^2+e\dot{e}) \; ? \; [1-(e-\dot{e})^2]^2 + (1-e^2)^{3/2} \},$$

(3d) $$I_3(e, \dot{e}, n=0) = 2\pi (1-e^2)^{-3/2} [1-(e-\dot{e})^2]^{-1/2} e^{-2} \dot{e}^{-2} (e-\dot{e})^{-1} \{ -e^2 (1-e^2)^{3/2} + [e^2 - e^4 - e^3\dot{e} - \dot{e}^2 + 2e^2\dot{e}^2 + \dot{e}^2(1-e^2)^{3/2}] [1-(e-\dot{e})^2]^{1/2} \},$$

(3e) $$I_4(e, \dot{e}, n=0) = 2\pi (1-e^2)^{-3/2} [1-(e-\dot{e})^2]^{-1/2} e^{-3} \dot{e}^{-2} (e-\dot{e})^{-2} \{ e^3(1-e^2)^{3/2} + (-e^3+e^5+e^4\dot{e}+3e\dot{e}^2-5e^3\dot{e}^2-2\dot{e}^3+3e^2\dot{e}^3)[1-(e-\dot{e})^2]^{1/2} + (-3e\dot{e}^2+2\dot{e}^3) \; ? \; (1-e^2)^{3/2}[1-(e-\dot{e})^2]^{1/2} \},$$

(3g) $$I_{0+}(e, \dot{e}, n=0) = 2\pi (1-e^2)^{-3/2} [1-(e-\dot{e})^2]^{-3/2} \dot{e}^{-3} \{ (2e^3-2e^5-3e^2\dot{e}+8e^4\dot{e}-12e^3\dot{e}^2+\dot{e}^3+8e^2\dot{e}^3-2e\dot{e}^4)(1-e^2)^{3/2} + (-2e^3+2e^5+3e^2\dot{e}-2e^4\dot{e}) \; ? \; [1-(e-\dot{e})^2]^{3/2} \},$$

(3h) $$I_{0-}(e, \dot{e}, n=0) = \pi (1-e^2)^{-5/2} [1-(e-\dot{e})^2]^{-1/2} \dot{e}^{-3} \{ (2e^3-4e^5+2e^7-6e^2\dot{e}+10e^4\dot{e}-4e^6\dot{e}+6e\dot{e}^2-5e^3\dot{e}^2+2e^5\dot{e}^2)[1-(e-\dot{e})^2]^{1/2} - 2(e-\dot{e})^3(1-e^2)^{5/2} \}.$$

Case $n = +1$

(4a) $$I_0(e, \dot{e}, n=+1) = 2\pi (e-\dot{e})^{-1/2} [1-(e-\dot{e})^2]^{-3/2} \dot{e}^{-2} \{ (-e^2+e^4-3e^3\dot{e}+\dot{e}^2+3e^2\dot{e}^2-e\dot{e}^3)(1-e^2)^{1/2} + e^2[1-(e-\dot{e})^2]^{3/2} \},$$

(4b) $$I_1(e, \dot{e}, n=+1) = 2\pi (1-e^2)^{-1/2} [1-(e-\dot{e})^2]^{-3/2} \dot{e}^{-2} \{ [e-(e-\dot{e})^3](1-e^2)^{1/2} - e[1-(e-\dot{e})^2]^{3/2} \},$$

(4c) $$I_2(e, \dot{e}, n=+1) = 2\pi (1-e^2)^{-1/2} [1-(e-\dot{e})^2]^{-3/2} \dot{e}^{-2} \{ (-1+e^2-3e\dot{e}+2\dot{e}^2) \; ? \; (1-e^2)^{1/2} + [1-(e-\dot{e})^2]^{3/2} \},$$

(4d) $$I_3(e, \dot{e}, n=+1) = 2\pi (1-e^2)^{-1/2} [1-(e-\dot{e})^2]^{-3/2} e^{-1} \dot{e}^{-2} (e-\dot{e})^{-2} \; ? \; \{ -(e-\dot{e})^2[1-(e-\dot{e})^2]^{3/2} + [e^2-e^4-2e\dot{e}+5e^3\dot{e}-7e^2\dot{e}^2+3e\dot{e}^3+\dot{e}^2[1-(e-\dot{e})^2]^{3/2}](1-e^2)^{1/2} \},$$

(4e) $$I_4(e, \dot{e}, n=+1) = 2\pi (1-e^2)^{-1/2} [1-(e-\dot{e})^2]^{-3/2} e^{-2} \dot{e}^{-2} (e-\dot{e})^{-3} \{ (e-\dot{e})^3 \; ? \; [1-(e-\dot{e})^2]^{3/2} + (-e^3+e^5+3e^2\dot{e}-6e^4\dot{e}+9e^3\dot{e}^2-4e^2\dot{e}^3)(1-e^2)^{1/2} + (-3e\dot{e}^2+\dot{e}^3)(1-e^2)^{1/2}[1-(e-\dot{e})^2]^{3/2} \},$$

(4g) $$I_{0+}(e, \dot{e}, n=+1) = \pi (1-e^2)^{-1/2} [1-(e-\dot{e})^2]^{-5/2} \dot{e}^{-3} \{ (-2e^3+4e^5-2e^7-10e^4\dot{e}+10e^6\dot{e}+5e^3\dot{e}^2-20e^5\dot{e}^2+2\dot{e}^3+5e^2\dot{e}^3+20e^4\dot{e}^3-5e\dot{e}^4-10e^3\dot{e}^4+\dot{e}^5+2e^2\dot{e}^5) \; ? \; (1-e^2)^{1/2} + 2e^3[1-(e-\dot{e})^2]^{5/2} \},$$

(4h) $$I_{0-}(e, \dot{e}, n=+1) = 2\pi (1-e^2)^{-3/2} [1-(e-\dot{e})^2]^{-3/2} \dot{e}^{-3} \{ (2e^3-2e^5-3e^2\dot{e}+8e^4\dot{e}$$

$- 12e^3 \dot{e}^2 + \dot{e}^3 + 8e^2 \dot{e}^3 - 2e \dot{e}^4) (1 - e^2)^{3/2} + (- 2e^3 + 2e^5 + 3e^2 \dot{e} - 2e^4 \dot{e})$
? $[1 - (e - \dot{e})^2]^{3/2} \}$.

Case $n = + 2$

(5a) $I_0(e, \dot{e}, n = + 2) = \pi [1 - (e - \dot{e})^2]^{-5/2} [2 + (e - \dot{e})^2]$,
(5b) $I_1(e, \dot{e}, n = + 2) = - 3\pi (e - \dot{e}) [1 - (e - \dot{e})^2]^{-5/2}$,
(5c) $I_2(e, \dot{e}, n = + 2) = \pi [1 + 2(e - \dot{e})^2] [1 - (e - \dot{e})^2]^{-5/2}$,
(5d) $I_3(e, \dot{e}, n = + 2) = \pi [1 - (e - \dot{e})^2]^{-5/2} (e - \dot{e})^{-3} \{ - 2 + 5e^2 - 6e^4 - 10e \dot{e} + 24e^3 \dot{e} + 5\dot{e}^2 - 36e^2 \dot{e}^2 + 24e \dot{e}^3 - 6\dot{e}^4 + 2[1 - (e - \dot{e})^2]^{5/2} \}$,
(5e) $I_4(e, \dot{e}, n = + 2) = \pi [1 - (e - \dot{e})^2]^{-5/2} (e - \dot{e})^{-4} \{ 6 - 15e^2 + 12e^4 + 30e \dot{e} - 48e^3 \dot{e}$
$- 15\dot{e}^2 + 72e^2 \dot{e}^2 - 48e \dot{e}^3 + 12 \dot{e}^4 - 6 [1 - (e - \dot{e})^2]^{5/2} \}$,
(5g) $I_{0+}(e, \dot{e}, n = + 2) = \pi [2 + 3(e - \dot{e})^2] [1 - (e - \dot{e})^2]^{-7/2}$,
(5h) $I_{0-}(e, \dot{e}, n = + 2) = I_{0+}(e, \dot{e}, n = + 1)$, (see formula (4g)) .

Case $n = + 3$

(6a) $I_0(e, \dot{e}, n = + 3) = \pi [1 - (e - \dot{e})^2]^{-7/2} (2 - e^2 - e^4 - 2e \dot{e} + 3e^3 \dot{e} + 3\dot{e}^2 - 3e^2 \dot{e}^2 + e \dot{e}^3)$,
(6b) $I_1(e, \dot{e}, n = + 3) = \pi [1 - (e - \dot{e})^2]^{-7/2} (e - \dot{e})^{-1} (- 3e^2 + 3e^4 + 7e \dot{e} - 8e^3 \dot{e} - 4\dot{e}^2 + 6e^2 \dot{e}^2 - \dot{e}^4)$,
(6c) $I_2(e, \dot{e}, n = + 3) = \pi [1 - (e - \dot{e})^2]^{-7/2} (1 + e^2 - 2e^4 - 5e \dot{e} + 6e^3 \dot{e} + 4\dot{e}^2 - 6e^2 \dot{e}^2 + 2e \dot{e}^3)$,
(6d) $I_3(e, \dot{e}, n = + 3) = \pi [1 - (e - \dot{e})^2]^{-7/2} (e - \dot{e})^{-4} \{ - 2e + 7e^3 - 11e^5 + 6e^7 - 14e^2 \dot{e} + 47e^4 \dot{e} - 34e^6 \dot{e} + 7e \dot{e}^2 - 78e^3 \dot{e}^2 + 78e^5 \dot{e}^2 + 62e^2 \dot{e}^3 - 90e^4 \dot{e}^3 - 23e \dot{e}^4 + 50e^3 \dot{e}^4 + 3\dot{e}^5 - 6e^2 \dot{e}^5 - 6e \dot{e}^6 + 2\dot{e}^7 + 2e [1 - (e - \dot{e})^2]^{7/2} \}$,
(6e) $I_4(e, \dot{e}, n = + 3) = \pi [1 - (e - \dot{e})^2]^{-7/2} (e - \dot{e})^{-5} \{ 6e - 21e^3 + 27e^5 - 12e^7 + 2\dot{e} + 35e^2 \dot{e} - 100e^4 \dot{e} + 64e^6 \dot{e} - 7e \dot{e}^2 + 130e^3 \dot{e}^2 - 132e^5 \dot{e}^2 - 7\dot{e}^3 - 60e^2 \dot{e}^3 + 120e^4 \dot{e}^3 - 5e \dot{e}^4 - 20e^3 \dot{e}^4 + 8\dot{e}^5 - 48e^2 \dot{e}^5 + 36e \dot{e}^6 - 8\dot{e}^7 + (- 6e - 2\dot{e}) [1 - (e - \dot{e})^2]^{7/2} \}$,
(6g) $I_{0+}(e, \dot{e}, n = + 3) = (\pi/4) [1 - (e - \dot{e})^2]^{-9/2} (e - \dot{e})^{-1} (8e + 4e^3 - 12e^5 - 8\dot{e} - 32e^2 \dot{e} + 45e^4 \dot{e} + 52e \dot{e}^2 - 60e^3 \dot{e}^2 - 24\dot{e}^3 + 30e^2 \dot{e}^3 - 3 \dot{e}^5)$,
(6h) $I_{0-}(e, \dot{e}, n = + 3) = \pi [2 + 3(e - \dot{e})^2] [1 - (e - \dot{e})^2]^{-7/2}$.

The above formulae must be considered under the restrictions $|e| < 1$, $|\dot{e}| < 1$ and $|e - \dot{e}| < 1$, which from a physical point of view guarantee that the trajectories of the gas particles are bounded (i.e., are ellipses) and do not intersect with each other. From a mathematical point of view these conditions also mean that the singularities in the expressions for metric, radius vector, Keplerian velocity and shear tensor (see [3], Appendix A), as well as in the integrals I_{0-}, I_{0+}, I_k (k = 0, 1, … , 4) are avoided. It should be pointed out that in some cases values $e = 0$, $\dot{e} = 0$ or $(e - \dot{e}) = 0$ may occur in the denominators of the expressions (2) - (6). Nevertheless, the integrals can also be analytically computed (even more easily !) by direct substitution of the so mentioned zero values into the original definitions of the integrals. These results may be compared with the limits derived from relations (2) - (6) when e, $\dot{e}$ or $(e - \dot{e})$ approach zero. The later calculations are based on the application of the L'Hospital's rule for solving of uncertainties of the type 0/0. In the both cases the final results are the same and consequently, we do not need to trouble about the nullification of the denominators - the transitions of the expressions (2) - (6) to the singular values of their arguments are continuous.

3. Auxiliary Functions and Dynamical Equation

According to paper [5], where the expressions for $Y(e, \dot{e}, n)$, $Z(e, \dot{e}, n)$ and $W(e, \dot{e}, n)$ are given in explicit form as linear combinations of the integrals $I_k(e, \dot{e}, n)$, (k = 0, 1, … , 4) (see formulae (2) - (4) from [5]). Having already the results (2) - (6) for integer n, we are in a position to compute $Y(e, \dot{e}, n)$, $Z(e, \dot{e}, n)$ and $W(e, \dot{e}, n)$ in a straightforward manner. There is not indispensable need to use the available linear relations between integrals I_{0-}, I_{0+} and I_k, (k = 0, 1, …, 4), in order to reduce the complexity of the initial formulae and, correspondingly, the intermediate calculations. Such simplifications are very desirable when the more general considerations of non-integer n are examined, when manifest evaluations of I_k
(k = 0, 1, … , 4) like (2) - (6) are not available. We shall directly write here the analytical form of the auxiliary functions $Y(e, \dot{e}, n)$, $Z(e, \dot{e}, n)$ and $W(e, \dot{e}, n)$ for $n = -1, 0, \ldots, 3$.

Case $n = -1$

(7a) $$3Y(e, \dot{e}, n = -1) = (p/GM)^{-1/2} (1 - e^2)^{-3/2} (3 - 3e^2 + 4e\dot{e}),$$

(7b) $$3Z(e, \dot{e}, n = -1) = (p/GM)^{-1/2} (1 - e^2)^{-1/2} [-1 + e^2 - 2e\dot{e} + 4(1 - e^2)^{1/2}]$$

(7c) $$9W(e, \dot{e}, n = -1) = (p/GM)^{-1/2} (1 - e^2)^{-3/2} e^{-2} [e^2 - 2e^4 + e^6 + 4e^3\dot{e} - 4e^5\dot{e} + 24\dot{e}^2 - 24e^2\dot{e}^2 + 4e^4\dot{e}^2 + (8e^2 - 24\dot{e}^2)(1 - e^2)^{3/2}].$$

Case $n = 0$

(8a) $$3Y(e, \dot{e}, n = 0) = (1 - e^2)^{-1/2} [1 - (e - \dot{e})^2]^{-1/2} \dot{e}^{-1} \{ (-3e + 3e^3 - \dot{e} - 5e^2\dot{e} + 2e\dot{e}^2) ? (1 - e^2)^{1/2} + (3e - 3e^3 + 4\dot{e} + 2e^2\dot{e}) [1 - (e - \dot{e})^2]^{1/2} \},$$

(8b) $3Z(e, \dot{e}, n = 0) = [1 - (e - \dot{e})^2]^{-1/2} \dot{e}^{-1} \{ -e + 2e^3 - e^5 - \dot{e} - 2e^2 \dot{e} + 3e^4 \dot{e} - 2e \dot{e}^2 - 2e^3 \dot{e}^2 + (e - e^3 + 2e^2 \dot{e})(1 - e^2)^{1/2} [1 - (e - \dot{e})^2]^{1/2} + 4\dot{e} [1 - (e - \dot{e})^2]^{1/2} \}$,

(8c) $9W(e, \dot{e}, n = 0) = (1 - e^2)^{-1/2} [1 - (e - \dot{e})^2]^{-1/2} e^{-1} \dot{e}^{-1} (e - \dot{e})^{-1} \{ (-e^3 + 2e^5 - e^7 + 2e^2 \dot{e} - 8e^4 \dot{e} + 6e^6 \dot{e} + 7e \dot{e}^2 + 2e^3 \dot{e}^2 - 13e^5 \dot{e}^2 + 12e^2 \dot{e}^3 + 12e^4 \dot{e}^3 - 8e \dot{e}^4 - 4e^3 \dot{e}^4) ? (1 - e^2)^{1/2} + (8e^2 \dot{e} - 8e \dot{e}^2 - 8\dot{e}^3)(1 - e^2)^{1/2} [1 - (e - \dot{e})^2]^{1/2} + (e^3 - 2e^5 + e^7 - e^2 \dot{e} + 6e^4 \dot{e} - 5e^6 \dot{e} - 8e \dot{e}^2 + 4e^3 \dot{e}^2 + 8e^5 \dot{e}^2 + 8\dot{e}^3 - 8e^2 \dot{e}^3 - 4e^4 \dot{e}^3) ? [1 - (e - \dot{e})^2]^{1/2} \}$.

<u>Case $n = +1$</u>

(9a) $3Y(e, \dot{e}, n = +1) = (p/GM)^{1/2} (1 - e^2)^{-1/2} [1 - (e - \dot{e})^2]^{-3/2} \dot{e}^{-1} \{ (4e - 8e^3 + 4e^5 + 3\dot{e} + 9e^2 \dot{e} - 12e^4 \dot{e} + 2e \dot{e}^2 + 12e^3 \dot{e}^2 - 4\dot{e}^3 - 4e^2 \dot{e}^3)(1 - e^2)^{1/2} + (-4e + 8e^3 - 4e^5 - 8e^2 \dot{e} + 8e^4 \dot{e} + 4e \dot{e}^2 - 4e^3 \dot{e}^2) [1 - (e - \dot{e})^2]^{1/2} \}$

(9b) $3Z(e, \dot{e}, n = +1) = (p/GM)^{1/2} [1 - (e - \dot{e})^2]^{-3/2} (e - \dot{e})^{-2} \{ -e^2 + 2e^4 - e^6 + 2e \dot{e} - 8e^3 \dot{e} + 6e^5 \dot{e} + 3\dot{e}^2 + 6e^2 \dot{e}^2 - 13e^4 \dot{e}^2 + 4e \dot{e}^3 + 12e^3 \dot{e}^3 - 4\dot{e}^4 - 4e^2 \dot{e}^4 + (4e^2 - 4e^4 - 8e \dot{e} + 16e^3 \dot{e} - 20e^2 \dot{e}^2 + 8e \dot{e}^3) [1 - (e - \dot{e})^2]^{1/2} \}$

(9c) $9W(e, \dot{e}, n = +1) = (p/GM)^{1/2} [1 - (e - \dot{e})^2]^{-3/2} \{ 1 - 2e^2 + e^4 + 4e \dot{e} - 4e^3 \dot{e} + 4e^2 \dot{e}^2 + (8 - 8e^2 + 16e \dot{e} - 8\dot{e}^2) [1 - (e - \dot{e})^2]^{1/2} \}$.

<u>Case $n = +2$</u>

(10a) $3Y(e, \dot{e}, n = +2) = (p/2GM) [1 - (e - \dot{e})^2]^{-5/2} (6 - 12e^2 + 6e^4 + 23e \dot{e} - 23e^3 \dot{e} - 9\dot{e}^2 + 31e^2 \dot{e}^2 - 14e \dot{e}^3)$,

(10b) $3Z(e, \dot{e}, n = +2) = (p/2GM) [1 - (e - \dot{e})^2]^{-5/2} (e - \dot{e})^{-3} \{ -2e^3 + 6e^5 - 6e^7 + 2e^9 + 6e^2 \dot{e} - 29e^4 \dot{e} + 40e^6 \dot{e} - 17e^8 \dot{e} + 18e \dot{e}^2 + 2e^3 \dot{e}^2 - 78e^5 \dot{e}^2 + 58e^7 \dot{e}^2 - 6\dot{e}^3 + 68e^2 \dot{e}^3 + 28e^4 \dot{e}^3 - 102e^6 \dot{e}^3 - 56e \dot{e}^4 + 74e^3 \dot{e}^4 + 98e^5 \dot{e}^4 + 9\dot{e}^5 - 84e^2 \dot{e}^5 - 49e^4 \dot{e}^5 + 26e \dot{e}^6 + 10e^3 \dot{e}^6 + (8e^3 - 24e^2 \dot{e}) [1 - (e - \dot{e})^2]^{5/2} \}$,

(10c) $9W(e, \dot{e}, n = 2) = (p/2GM) [1 - (e - \dot{e})^2]^{-5/2} (e - \dot{e})^{-3} \{ 2e^3 - 6e^5 + 6e^7 - 2e^9 - 6e^2 \dot{e} + 27e^4 \dot{e} - 36e^6 \dot{e} + 15e^8 \dot{e} + 6e \dot{e}^2 - 44e^3 \dot{e}^2 + 82e^5 \dot{e}^2 - 44e^7 \dot{e}^2 - 18\dot{e}^3 + 70e^2 \dot{e}^3 - 110e^4 \dot{e}^3 + 62e^6 \dot{e}^3 - 86e \dot{e}^4 + 132e^3 \dot{e}^4 - 38e^5 \dot{e}^4 + 39\dot{e}^5 - 142e^2 \dot{e}^5 - e^4 \dot{e}^5 + 92e \dot{e}^6 + 12e^3 \dot{e}^6 - 24\dot{e}^7 - 4e^2 \dot{e}^7 + (16e^3 - 32e^5 + 16e^7 - 48e^2 \dot{e} + 160e^4 \dot{e} - 112e^6 \dot{e} + 48e \dot{e}^2 - 320e^3 \dot{e}^2 + 336e^5 \dot{e}^2 + 288e^2 \dot{e}^3 - 544e^4 \dot{e}^3 - 96e \dot{e}^4 + 496e^3 \dot{e}^4 - 240e^2 \dot{e}^5 + 48e \dot{e}^6) [1 - (e - \dot{e})^2]^{1/2} \}$.

<u>Case $n = +3$</u>

(11a) $3Y(e, \dot{e}, n = +3) = (1/2)(p/GM)^{3/2} [1 - (e - \dot{e})^2]^{-7/2} (6 - 18e^2 + 18e^4 - 6e^6 + 30e \dot{e} - 60e^3 \dot{e} + 30e^5 \dot{e} - 7\dot{e}^2 + 65e^2 \dot{e}^2 - 58e^4 \dot{e}^2 - 20e \dot{e}^3 + 50e^3 \dot{e}^3 - 4\dot{e}^4 - 16e^2 \dot{e}^4)$, (11b) $3Z(e, \dot{e}, n = +3) = (1/2)\,(p/GM)^{3/2}\,[1 - (e - \dot{e})^2]^{-7/2}\,(e - \dot{e})^{-4}\,\{ -2e^4 + 8e^6 - 12e^8 + 8e^{10} - 2e^{12} + 8e^3 \dot{e} - 46e^5 \dot{e} + 90e^7 \dot{e} - 74e^9 \dot{e} + 22e^{11} \dot{e} + 36e^2 \dot{e}^2 - 43e^4 \dot{e}^2 - 124e^6 \dot{e}^2 + 233e^8 \dot{e}^2 - 102e^{10} \dot{e}^2 - 24e \dot{e}^3 + 248e^3 \dot{e}^3 - 206e^5 \dot{e}^3 - 280e^7 \dot{e}^3 + 262e^9 \dot{e}^3 + 6\dot{e}^4 - 230e^2 \dot{e}^4 + 680e^4 \dot{e}^4 - 66e^6 \dot{e}^4 - 410e^8 \dot{e}^4 + 70e \dot{e}^5 - 658e^3 \dot{e}^5 + 546e^5 \dot{e}^5 + 402e^7 \dot{e}^5 - 7\dot{e}^6 + 260e^2 \dot{e}^6 - 607e^4 \dot{e}^6 - 242e^6 \dot{e}^6 - 26e \dot{e}^7 + 296e^3 \dot{e}^7 + 82e^5 \dot{e}^7 - 4\dot{e}^8 - 56e^2 \dot{e}^8 - 12e^4 \dot{e}^8 + (8e^4 - 24e^6 + 24e^8 - 8e^{10} - 32e^3 \dot{e} + 144e^5 \dot{e} - 192e^7 \dot{e} + 80e^9 \dot{e} - 216e^4 \dot{e}^2 + 528e^6 \dot{e}^2 - 312e^8 \dot{e}^2 + 96e^3 \dot{e}^3 - 672e^5 \dot{e}^3 + 640e^7 \dot{e}^3 + 408e^4 \dot{e}^4 - 760e^6 \dot{e}^4 - 96e^3 \dot{e}^5 + 528e^5 \dot{e}^5 - 200e^4 \dot{e}^6 + 32e^3 \dot{e}^7)\,[1 - (e - \dot{e})^2]^{1/2} \}$,

(11c) $9W(e, \dot{e}, n = +3) = (1/2)\,(p/GM)^{3/2}\,[1 - (e - \dot{e})^2]^{-7/2}\,(e - \dot{e})^{-4}\,\{ 2e^4 - 8e^6 + 12e^8 - 8e^{10} + 2e^{12} - 8e^3 \dot{e} + 42e^5 \dot{e} - 78e^7 \dot{e} + 62e^9 \dot{e} - 18e^{11} \dot{e} + 28e^2 \dot{e}^2 - 133e^4 \dot{e}^2 + 248e^6 \dot{e}^2 - 209e^8 \dot{e}^2 + 66e^{10} \dot{e}^2 - 72e \dot{e}^3 + 384e^3 \dot{e}^3 - 662e^5 \dot{e}^3 + 472e^7 \dot{e}^3 - 122e^9 \dot{e}^3 + 18\dot{e}^4 - 542e^2 \dot{e}^4 + 1340e^4 \dot{e}^4 - 914e^6 \dot{e}^4 + 102e^8 \dot{e}^4 + 302e \dot{e}^5 - 1642e^3 \dot{e}^5 + 1474e^5 \dot{e}^5 + 18e^7 \dot{e}^5 - 45\dot{e}^6 + 1088e^2 \dot{e}^6 - 1681e^4 \dot{e}^6 - 122e^6 \dot{e}^6 - 338e \dot{e}^7 + 1200e^3 \dot{e}^7 + 114e^5 \dot{e}^7 + 32\dot{e}^8 - 476e^2 \dot{e}^8 - 48e^4 \dot{e}^8 + 80e \dot{e}^9 + 8e^3 \dot{e}^9 + (16e^4 - 64e^3 \dot{e} + 80e^2 \dot{e}^2)\,[1 - (e - \dot{e})^2]^{7/2} \}$.

Let us remind some of the notations used above: p is the focal parameter of the ellipse (for circular orbits p is simply the radius of the particle trajectory at the considered moment), $u \equiv \ln p$, $\dot{e} \equiv \partial e/\partial u$, G is the Newton's gravitational constant and M is the mass of the compact object around which the accretion disc rotates. The knowledge of the factor $(p/GM)^{n/2}$ (for all astrophysically significant values of the exponent n) is not needed, because after the substitution of the auxiliary functions Y, Z and W into the dynamical equation (1), this factor cancels out. The partial derivatives of these auxiliary functions with respect to e and $\dot{e}$ are computable without any technical problems and we shall not give here their analytical evaluations. We shall note again that the singularity problem which may arise in relation to null values of e, $\dot{e}$ and $(e - \dot{e})$ in the denominators can be overcome by means of the L'Hospital's theorem (indeterminations of the 0/0 type). The same observation will hold later for the coefficients of dynamical equation (1). Here, another property of the free term of this equation should be mentioned. Upon computing in explicit form the expression (3Y) [9W - 2(3Z) + $(e^2 - 1)$ (3Y)], substituting it into equation (1), and reducing it to a common denominator with the second term on the left-hand side of (1), it turns out that the result factorizes with respect to $\dot{e}$. The free term is absorbed into the term containing the first derivatives of $\dot{e}$ and the dynamical equation

(1) becomes a second order *homogeneous* differential equation. It may be shown that this feature is inherent to (1) not only for the considered integer values of $n = -1, 0, \ldots, +3$, but also for an arbitrary physically acceptable n. In view of this, we rewrite equation (1) in the following form:

(12) $\quad \mathrm{A}(e, \dot{e}, n)\, \ddot{e} + \mathrm{B}(e, \dot{e}, n)\, \dot{e} = 0\,.$

We can write the solution of the above equation as

(13) $\quad e(u, n) = \dot{e}_0 \int exp\, \{- \int [\, \mathrm{B}(e, \dot{e}, n)/\mathrm{A}(e, \dot{e}, n)\,]\, du\} + e_0\,,$

where, according to the general theory of second order ordinary differential equations, the solution (for a given value of the exponent n) depends on two integration constants $e_0 \equiv e(u_0, n)$ and $\dot{e}_0 \equiv \dot{e}(u_0, n)$; $u_0 \equiv ln\, p_0$ is a fixed initial value. For example, p_0 may be the focal parameter of the innermost/outermost ellipse bounding the disc.

The above formally written solution is not useful because $\mathrm{A}(e, \dot{e}, n)$ and $\mathrm{B}(e, \dot{e}, n)$ are known in an explicit form as functions on e, $\dot{e}$ and n, but not as functions on u (so far $e = e(u, n)$ is not solved yet !). A method for solving the equation (11) by means of expanding the eccentricity e by powers in u will be considered in a forthcoming paper. Now we shall restrict ourselves only to give the explicit form of dynamical equation (11) for two values of n, namely $n = -1$ and $n = +2$. For the other considered integer values of n ($n = 0, +1$ and $+3$), the analytical expressions are too long to be given. For this reason, we depict graphically the dependencies of the ratios of the coefficients of equation (11) A and B on e and $\dot{e}$ for fixed n.

Dynamical equation : Case $n = -1$

(14) $\quad 2e^3 (1 - e^2)^{3/2} [\, 8 + (1 - e^2)^{1/2}\,]\, \ddot{e} + [\, -3e^3 + 6e^5 - 3e^7 - 36\dot{e} + 74e^2 \dot{e} - 46e^4 \dot{e} + 8e^6 \dot{e} - 48e\, \dot{e}^2 + 48e^3 \dot{e}^2 - 8e^5 \dot{e}^2 + (\, 12e^3 - 12e^5 + 36\dot{e} - 56e^2 \dot{e} + 68e^4 \dot{e} + 48e\, \dot{e}^2 - 48e^3 \dot{e}^2\,)\, (1 - e^2)^{1/2}\,]\, \dot{e} = 0\,.$

Dynamical equation : Case $n = +2$

(15) $\quad \{ -10e^4 + 40e^6 - 60e^8 + 40e^{10} - 10e^{12} + 144e\, \dot{e} - 560e^3 \dot{e} + 724e^5 \dot{e} - 252e^7 \dot{e} - 148e^9 \dot{e} + 92e^{11} \dot{e} + 1104e^2 \dot{e}^2 - 3165e^4 \dot{e}^2 + 2654e^6 \dot{e}^2 - 229e^8 \dot{e}^2 - 364e^{10} \dot{e}^2 - 504e\, \dot{e}^3 + 4628e^3 \dot{e}^3 - 7208e^5 \dot{e}^3 + 2272e^7 \dot{e}^3 + 812e^9 \dot{e}^3 + 18\dot{e}^4 - 2842e^2 \dot{e}^4 + 9366e^4 \dot{e}^4 - 5554e^6 \dot{e}^4 - 1120e^8 \dot{e}^4 + 696e\, \dot{e}^5 - 6404e^3 \dot{e}^5 + 7016e^5 \dot{e}^5 + 980e^7 \dot{e}^5 - 81\dot{e}^6 + 2258e^2 \dot{e}^6 - 5021e^4 \dot{e}^6 -$

$$
\begin{gathered}
532e^6 \dot{e}^6 - 408e \dot{e}^7 + 1940e^3 \dot{e}^7 + 164e^5 \dot{e}^7 + 54\dot{e}^8 - 316e^2 \dot{e}^8 - 22e^4 \dot{e}^8 + \\
(28e^4 - 84e^6 + 84e^8 - 28e^{10} - 144e \dot{e} + 416e^3 \dot{e} - 228e^5 \dot{e} \\
- 216e^7 \dot{e} + 172e^9 \dot{e} - 852e^2 \dot{e}^2 + 1572e^4 e^2 - 300e^6 \dot{e}^2 - 420e^8 \dot{e}^2 + 360e \\
\dot{e}^3 - 2328e^3 \dot{e}^3 + 1644e^5 \dot{e}^3 + 500e^7 \dot{e}^3 + 1176e^2 \dot{e}^4 - 2124e^4 \dot{e}^4 - 260e^6 \\
\dot{e}^4 - 108e \dot{e}^5 + 984 e^3 \dot{e}^5 - 12e^5 \dot{e}^5 + 36e^2 \dot{e}^6 + 68e^4 \dot{e}^6 - 108e \dot{e}^7 - 20e^3 \\
\dot{e}^7) [1 - (e - \dot{e})^2]^{1/2} \} \ddot{e} + \{ 6e^4 - 24e^6 + 36e^8 - 24e^{10} + 6e^{12} - 72e \dot{e} + \\
272e^3 \dot{e} - 323e^5 \dot{e} + 57e^7 \dot{e} + 127e^9 \dot{e} - 61e^{11} \dot{e} - 72\dot{e}^2 - 236e^2 \dot{e}^2 + 982e^4 \\
\dot{e}^2 - 683e^6 \dot{e}^2 - 276e^8 \dot{e}^2 + 285e^{10} \dot{e}^2 - 360e \dot{e}^3 - 234e^3 \dot{e}^3 + 889e^5 \dot{e}^3 + \\
524e^7 \dot{e}^3 - 819e^9 \dot{e}^3 + 270\dot{e}^4 - 1528e^2 \dot{e}^4 + 1255e^4 \dot{e}^4 - 1538e^6 \dot{e}^4 + \\
1617e^8 \dot{e}^4 + 1493e \dot{e}^5 - 4141e^3 \dot{e}^5 + 3717e^5 \dot{e}^5 - 2289e^7 \dot{e}^5 - 366\dot{e}^6 + \\
4043e^2 \dot{e}^6 - 5488e^4 \dot{e}^6 + 2331e^6 \dot{e}^6 - 1717e \dot{e}^7 + 4870e^3 \dot{e}^7 \\
- 1665e^5 \dot{e}^7 + 261\dot{e}^8 - 2550e^2 \dot{e}^8 + 789e^4 \dot{e}^8 + 722e \dot{e}^9 - 222e^3 \dot{e}^9 - 84\dot{e}^{10} \\
+ 28e^2 \dot{e}^{10} + [- 24e^4 + 72e^6 - 72e^8 + 24e^{10} + 72e \dot{e} - 164e^3 \dot{e} - 72 e^5 \dot{e} \\
+ 348e^7 \dot{e} - 184e^9 \dot{e} + 72\dot{e}^2 + 92e^2 \dot{e}^2 + 12e^4 \dot{e}^2 - 828e^6 \dot{e}^2 + 652e^8 \dot{e}^2 + \\
396e \dot{e}^3 - 756e^3 \dot{e}^3 + 1632e^5 \dot{e}^3 - 1416e^7 \dot{e}^3 - 252\dot{e}^4 + 1788e^2 \dot{e}^4 - \\
2868e^4 \dot{e}^4 + 2060e^6 \dot{e}^4 - 1332e \dot{e}^5 + 3492e^3 \dot{e}^5 - 2024e^5 \dot{e}^5 + 288\dot{e}^6 - \\
2460e^2 \dot{e}^6 + 1284e^4 \dot{e}^6 + 864e \dot{e}^7 - 472e^3 \dot{e}^7 - 108\dot{e}^8 + 76e^2 \dot{e}^8] [1 - (e - \\
\dot{e})^2]^{1/2} \} \dot{e} = 0 .
\end{gathered}
$$

4. Conclusions

The complexity of the accretion flows phenomena requires using both analytical and numerical approaches for their description. The analytical methods are preferable because of the compact representation of the results, suitable for their interpretations and further applications. They also allow to control more clearly the process of derivation of the solutions and the influence of the accepted approximations on the output data. It often happens that the analytical treatment of the considered problem is not possible to be performed up to the final stage of the computational process and further use of numerical methods is needed. Nevertheless, even this partial application of the analytical description reveals which of the approximations are more important and suggests more effectively how to overcome them and how to improve the model without complicating it too much. Of course, the comparisons with astronomical observations, in our case, observational data of close binary systems containing accretion discs, serve as a test for the task's successful solution. As mentioned above, already existing theoretical models of eccentric discs around the compact stars in binaries explain successfully many of the observed properties of these astronomical objects. For example, Murray [7] has compared the theoretical predictions for the precession rates of eccentric discs with the observed superhump periods. It was found that the inclusion of a retrograde pressure contribution improves the fit to the data and the consistency with the suggestion that the eccentricity is generated at the 3:1 Lindblad resonance.

It may be supposed that the detailed analytical treatment of the more simple model developed by Lyubarskij et al. [3] would be suggestive for finding analytical solutions to more complicated models like that worked out by Ogilvie [4]. The difficulties and limitations inherent to attempts to resolve the more simplified (and probably easier to solve!) problem would also be indicators of how perspective are the efforts to attain analogous progress in the investigation of the complicated situation. The true answer of this puzzle is expected to be achieved through a series of improved step-by-step analytical and numerical evaluations of the particular accretion disc models.

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ТЪНКИ ВИСКОЗНИ ЕЛИПТИЧНИ АКРЕЦИОННИ ДИСКОВЕ С ОРБИТИ ИМАЩИ ОБЩА ДЪЛЖИНА НА ПЕРИАСТРОНА. ДИНАМИЧНО УРАВНЕНИЕ ЗА ЦЕЛОЧИСЛЕНИ СТОЙНОСТИ НА СТЕПЕНИТЕ В ЗАКОНА ЗА ВИСКОЗИТЕТА

Димитър Димитров

Резюме

Ние разглеждаме модел на тънък стационарен вискозен акреционен диск около компактен обект със звездна маса, разработен от Любарски и др. [3]. Орбитите на газовите частици са елипси чиито ексцентрицитети могат да варират от вътрешната към външната част на диска и чиито апсидни линии лежат върху една права. Приетият коефициент за вискозитета η удовлетворява зависимостта $\eta = \beta \Sigma^n$, със Σ - повърхностната плътност на акреционния диск, β и n - константи. Нашите разглеждания третират случаите когато експонентата n приема целочислени стойности, а именно n = - 1, 0, 1, 2 и 3, които лежат във физически обоснована област. Ние получаваме в явен вид спомагателните функции въведени от Любарски и др. За две стойности на n = - 1 и n = + 2 ние също сме написали в явен вид динамичното уравнение обуславящо радиалната структура на диска. За другите случаи ние се ограничаваме с графични представяния на отношенията на коефициентите на това уравнение.